```
int[n_] := Integrate[1 / (x^n + 1), x]
```

```
For[i = 1, i ≤ 10, i++,
 Print["=== Integrate[1/(x^", i, "+1),x] = \n", TextForm[int[i]]]
]
```

=== Integrate[1/(x^1+1),x] =

$$\text{Log}[1+x]$$

=== Integrate[1/(x^2+1),x] =

$$\text{ArcTan}[x]$$

=== Integrate[1/(x^3+1),x] =

$$\frac{\text{ArcTan}\left[\frac{-1+2x}{\sqrt{3}}\right]}{\sqrt{3}}+\frac{1}{3}\text{Log}[1+x]-\frac{1}{6}\text{Log}\left[1-x+x^2\right]$$

=== Integrate[1/(x^4+1),x] =

$$\frac{-2\,\text{ArcTan}\left[1-\sqrt{2}\,x\right]+2\,\text{ArcTan}\left[1+\sqrt{2}\,x\right]-\text{Log}\left[1-\sqrt{2}\,x+x^2\right]+\text{Log}\left[1+\sqrt{2}\,x+x^2\right]}{4\sqrt{2}}$$

=== Integrate[1/(x^5+1),x] =

$$\frac{1}{20}\left(-2\sqrt{10-2\sqrt{5}}\,\text{ArcTan}\left[\frac{1+\sqrt{5}-4x}{\sqrt{10-2\sqrt{5}}}\right]+2\sqrt{2\left(5+\sqrt{5}\right)}\,\text{ArcTan}\left[\frac{-1+\sqrt{5}+4x}{\sqrt{2\left(5+\sqrt{5}\right)}}\right]+\right.$$
$$\left.4\,\text{Log}[1+x]+\left(-1+\sqrt{5}\right)\text{Log}\left[1+\frac{1}{2}\left(-1+\sqrt{5}\right)x+x^2\right]-\left(1+\sqrt{5}\right)\text{Log}\left[1-\frac{1}{2}\left(1+\sqrt{5}\right)x+x^2\right]\right)$$

=== Integrate[1/(x^6+1),x] =

$$\frac{1}{12}\left(-2\,\text{ArcTan}\left[\sqrt{3}-2x\right]+4\,\text{ArcTan}[x]+\right.$$
$$\left.2\,\text{ArcTan}\left[\sqrt{3}+2x\right]-\sqrt{3}\,\text{Log}\left[1-\sqrt{3}\,x+x^2\right]+\sqrt{3}\,\text{Log}\left[1+\sqrt{3}\,x+x^2\right]\right)$$

=== Integrate[1/(x^7+1),x] =

$$\frac{2}{7}\text{ArcTan}\left[\text{Sec}\left[\frac{\pi}{14}\right]\left(x-\text{Sin}\left[\frac{\pi}{14}\right]\right)\right]\text{Cos}\left[\frac{\pi}{14}\right]+\frac{2}{7}\text{ArcTan}\left[\text{Sec}\left[\frac{3\pi}{14}\right]\left(x+\text{Sin}\left[\frac{3\pi}{14}\right]\right)\right]\text{Cos}\left[\frac{3\pi}{14}\right]+$$
$$\frac{1}{7}\text{Log}[1+x]-\frac{1}{7}\text{Cos}\left[\frac{\pi}{7}\right]\text{Log}\left[1+x^2-2x\,\text{Cos}\left[\frac{\pi}{7}\right]\right]-\frac{1}{7}\text{Log}\left[1+x^2-2x\,\text{Sin}\left[\frac{\pi}{14}\right]\right]\text{Sin}\left[\frac{\pi}{14}\right]+$$
$$\frac{2}{7}\text{ArcTan}\left[\left(x-\text{Cos}\left[\frac{\pi}{7}\right]\right)\text{Csc}\left[\frac{\pi}{7}\right]\right]\text{Sin}\left[\frac{\pi}{7}\right]+\frac{1}{7}\text{Log}\left[1+x^2+2x\,\text{Sin}\left[\frac{3\pi}{14}\right]\right]\text{Sin}\left[\frac{3\pi}{14}\right]$$

=== Integrate[1/(x^8+1),x] =

$$\frac{1}{4}\text{ArcTan}\left[\text{Sec}\left[\frac{\pi}{8}\right]\left(x-\text{Sin}\left[\frac{\pi}{8}\right]\right)\right]\text{Cos}\left[\frac{\pi}{8}\right]+\frac{1}{4}\text{ArcTan}\left[\text{Sec}\left[\frac{\pi}{8}\right]\left(x+\text{Sin}\left[\frac{\pi}{8}\right]\right)\right]\text{Cos}\left[\frac{\pi}{8}\right]-$$
$$\frac{1}{8}\text{Cos}\left[\frac{\pi}{8}\right]\text{Log}\left[1+x^2-2x\,\text{Cos}\left[\frac{\pi}{8}\right]\right]+\frac{1}{8}\text{Cos}\left[\frac{\pi}{8}\right]\text{Log}\left[1+x^2+2x\,\text{Cos}\left[\frac{\pi}{8}\right]\right]+$$
$$\frac{1}{4}\text{ArcTan}\left[\left(x-\text{Cos}\left[\frac{\pi}{8}\right]\right)\text{Csc}\left[\frac{\pi}{8}\right]\right]\text{Sin}\left[\frac{\pi}{8}\right]+\frac{1}{4}\text{ArcTan}\left[\left(x+\text{Cos}\left[\frac{\pi}{8}\right]\right)\text{Csc}\left[\frac{\pi}{8}\right]\right]\text{Sin}\left[\frac{\pi}{8}\right]-$$
$$\frac{1}{8}\text{Log}\left[1+x^2-2x\,\text{Sin}\left[\frac{\pi}{8}\right]\right]\text{Sin}\left[\frac{\pi}{8}\right]+\frac{1}{8}\text{Log}\left[1+x^2+2x\,\text{Sin}\left[\frac{\pi}{8}\right]\right]\text{Sin}\left[\frac{\pi}{8}\right]$$

=== Integrate[1/(x^9+1),x] =

$$\frac{1}{18}\Bigg(2\sqrt{3}\,\text{ArcTan}\Big[\frac{-1+2\,x}{\sqrt{3}}\Big]+4\,\text{ArcTan}\Big[x\,\text{Sec}\Big[\frac{\pi}{18}\Big]+\text{Tan}\Big[\frac{\pi}{18}\Big]\Big]\,\text{Cos}\Big[\frac{\pi}{18}\Big]+2\,\text{Log}[1+x]-\text{Log}\big[1-x+x^2\big]-2\,\text{Cos}\Big[\frac{\pi}{9}\Big]\,\text{Log}\Big[1+x^2-2\,x\,\text{Cos}\Big[\frac{\pi}{9}\Big]\Big]+2\,\text{Cos}\Big[\frac{2\,\pi}{9}\Big]\,\text{Log}\Big[1+x^2+2\,x\,\text{Cos}\Big[\frac{2\,\pi}{9}\Big]\Big]+2\,\text{Log}\Big[1+x^2+2\,x\,\text{Sin}\Big[\frac{\pi}{18}\Big]\Big]\,\text{Sin}\Big[\frac{\pi}{18}\Big]-4\,\text{ArcTan}\Big[\text{Cot}\Big[\frac{\pi}{9}\Big]-x\,\text{Csc}\Big[\frac{\pi}{9}\Big]\Big]\,\text{Sin}\Big[\frac{\pi}{9}\Big]+4\,\text{ArcTan}\Big[\Big(x+\text{Cos}\Big[\frac{2\,\pi}{9}\Big]\Big)\,\text{Csc}\Big[\frac{2\,\pi}{9}\Big]\Big]\,\text{Sin}\Big[\frac{2\,\pi}{9}\Big]\Bigg)$$

=== Integrate[1/(x^10+1),x] =

$$\frac{1}{40}\Bigg(2\left(-1+\sqrt{5}\right)\text{ArcTan}\Bigg[\frac{\sqrt{2\left(5+\sqrt{5}\right)}-4\,x}{1-\sqrt{5}}\Bigg]+8\,\text{ArcTan}[x]+2\left(1+\sqrt{5}\right)\text{ArcTan}\Bigg[\frac{-\sqrt{10-2\sqrt{5}}+4\,x}{1+\sqrt{5}}\Bigg]+2\left(1+\sqrt{5}\right)\text{ArcTan}\Bigg[\frac{\sqrt{10-2\sqrt{5}}+4\,x}{1+\sqrt{5}}\Bigg]+2\left(-1+\sqrt{5}\right)\text{ArcTan}\Bigg[\frac{\sqrt{2\left(5+\sqrt{5}\right)}+4\,x}{-1+\sqrt{5}}\Bigg]-\sqrt{10-2\sqrt{5}}\,\text{Log}\Bigg[1-\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\,x+x^2\Bigg]+\sqrt{10-2\sqrt{5}}\,\text{Log}\Bigg[1+\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\,x+x^2\Bigg]-\sqrt{2\left(5+\sqrt{5}\right)}\,\text{Log}\Bigg[1-\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\,x+x^2\Bigg]+\sqrt{2\left(5+\sqrt{5}\right)}\,\text{Log}\Bigg[1+\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\,x+x^2\Bigg]\Bigg)$$

```
For[i = 1, i ≤ 10, i++,
 Print["=== Integrate[1/(x^", i, "+1),x] = \n", InputForm[int[i]]]
]
```

```
=== Integrate[1/(x^1+1),x] =
Log[1 + x]

=== Integrate[1/(x^2+1),x] =
ArcTan[x]

=== Integrate[1/(x^3+1),x] =
ArcTan[(-1 + 2 * x) / Sqrt[3]] / Sqrt[3] + Log[1 + x] / 3 - Log[1 - x + x^2] / 6

=== Integrate[1/(x^4+1),x] =
(-2 * ArcTan[1 - Sqrt[2] * x] + 2 * ArcTan[1 + Sqrt[2] * x] -
    Log[1 - Sqrt[2] * x + x^2] + Log[1 + Sqrt[2] * x + x^2]) / (4 * Sqrt[2])

=== Integrate[1/(x^5+1),x] =
(-2 * Sqrt[10 - 2 * Sqrt[5]] * ArcTan[(1 + Sqrt[5] - 4 * x) / Sqrt[10 - 2 * Sqrt[5]]] +
    2 * Sqrt[2 * (5 + Sqrt[5])] * ArcTan[(-1 + Sqrt[5] + 4 * x) / Sqrt[2 * (5 + Sqrt[5])]] +
    4 * Log[1 + x] + (-1 + Sqrt[5]) * Log[1 + ((-1 + Sqrt[5]) * x) / 2 + x^2] -
    (1 + Sqrt[5]) * Log[1 - ((1 + Sqrt[5]) * x) / 2 + x^2]) / 20

=== Integrate[1/(x^6+1),x] =
(-2 * ArcTan[Sqrt[3] - 2 * x] + 4 * ArcTan[x] + 2 * ArcTan[Sqrt[3] + 2 * x] -
    Sqrt[3] * Log[1 - Sqrt[3] * x + x^2] + Sqrt[3] * Log[1 + Sqrt[3] * x + x^2]) / 12

=== Integrate[1/(x^7+1),x] =
(2 * ArcTan[Sec[Pi / 14] * (x - Sin[Pi / 14])] * Cos[Pi / 14]) / 7 +
  (2 * ArcTan[Sec[(3 * Pi) / 14] * (x + Sin[(3 * Pi) / 14])] * Cos[(3 * Pi) / 14]) / 7 +
  Log[1 + x] / 7 - (Cos[Pi / 7] * Log[1 + x^2 - 2 * x * Cos[Pi / 7]]) / 7 -
  (Log[1 + x^2 - 2 * x * Sin[Pi / 14]] * Sin[Pi / 14]) / 7 +
  (2 * ArcTan[(x - Cos[Pi / 7]) * Csc[Pi / 7]] * Sin[Pi / 7]) / 7 +
  (Log[1 + x^2 + 2 * x * Sin[(3 * Pi) / 14]] * Sin[(3 * Pi) / 14]) / 7

=== Integrate[1/(x^8+1),x] =
(ArcTan[Sec[Pi / 8] * (x - Sin[Pi / 8])] * Cos[Pi / 8]) / 4 +
  (ArcTan[Sec[Pi / 8] * (x + Sin[Pi / 8])] * Cos[Pi / 8]) / 4 -
  (Cos[Pi / 8] * Log[1 + x^2 - 2 * x * Cos[Pi / 8]]) / 8 +
  (Cos[Pi / 8] * Log[1 + x^2 + 2 * x * Cos[Pi / 8]]) / 8 +
  (ArcTan[(x - Cos[Pi / 8]) * Csc[Pi / 8]] * Sin[Pi / 8]) / 4 +
  (ArcTan[(x + Cos[Pi / 8]) * Csc[Pi / 8]] * Sin[Pi / 8]) / 4 -
  (Log[1 + x^2 - 2 * x * Sin[Pi / 8]] * Sin[Pi / 8]) / 8 +
  (Log[1 + x^2 + 2 * x * Sin[Pi / 8]] * Sin[Pi / 8]) / 8

=== Integrate[1/(x^9+1),x] =
(2 * Sqrt[3] * ArcTan[(-1 + 2 * x) / Sqrt[3]] +
    4 * ArcTan[x * Sec[Pi / 18] + Tan[Pi / 18]] * Cos[Pi / 18] + 2 * Log[1 + x] -
    Log[1 - x + x^2] - 2 * Cos[Pi / 9] * Log[1 + x^2 - 2 * x * Cos[Pi / 9]] +
    2 * Cos[(2 * Pi) / 9] * Log[1 + x^2 + 2 * x * Cos[(2 * Pi) / 9]] +
    2 * Log[1 + x^2 + 2 * x * Sin[Pi / 18]] * Sin[Pi / 18] -
    4 * ArcTan[Cot[Pi / 9] - x * Csc[Pi / 9]] * Sin[Pi / 9] +
    4 * ArcTan[(x + Cos[(2 * Pi) / 9]) * Csc[(2 * Pi) / 9]] * Sin[(2 * Pi) / 9]) / 18

=== Integrate[1/(x^10+1),x] =

 (2 * (-1 + Sqrt[5]) * ArcTan[(Sqrt[2 * (5 + Sqrt[5])] - 4 * x) / (1 - Sqrt[5])] + 8 * ArcTan[
      x] + 2 * (1 + Sqrt[5]) * ArcTan[(-Sqrt[10 - 2 * Sqrt[5]] + 4 * x) / (1 + Sqrt[5])] +
    2 * (1 + Sqrt[5]) * ArcTan[(Sqrt[10 - 2 * Sqrt[5]] + 4 * x) / (1 + Sqrt[5])] +
    2 * (-1 + Sqrt[5]) * ArcTan[(Sqrt[2 * (5 + Sqrt[5])] + 4 * x) / (-1 + Sqrt[5])] -
    Sqrt[10 - 2 * Sqrt[5]] * Log[1 - Sqrt[(5 - Sqrt[5]) / 2] * x + x^2] +
    Sqrt[10 - 2 * Sqrt[5]] * Log[1 + Sqrt[(5 - Sqrt[5]) / 2] * x + x^2] -
    Sqrt[2 * (5 + Sqrt[5])] * Log[1 - Sqrt[(5 + Sqrt[5]) / 2] * x + x^2] +
    Sqrt[2 * (5 + Sqrt[5])] * Log[1 + Sqrt[(5 + Sqrt[5]) / 2] * x + x^2]) / 40
```

```
For[i = 1, i ≤ 10, i++,
 Print["=== Integrate[1/(x^", i, "+1),x] = \n", TraditionalForm[int[i]]]
]
```

```
=== Integrate[1/(x^1+1),x] =
```
$\log(x+1)$

```
=== Integrate[1/(x^2+1),x] =
```
$\tan^{-1}(x)$

```
=== Integrate[1/(x^3+1),x] =
```
$$-\frac{1}{6}\log\left(x^2-x+1\right)+\frac{1}{3}\log(x+1)+\frac{\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
=== Integrate[1/(x^4+1),x] =
```
$$\frac{1}{4\sqrt{2}}\left(-\log\left(x^2-\sqrt{2}\,x+1\right)+\log\left(x^2+\sqrt{2}\,x+1\right)-2\tan^{-1}\left(1-\sqrt{2}\,x\right)+2\tan^{-1}\left(\sqrt{2}\,x+1\right)\right)$$

```
=== Integrate[1/(x^5+1),x] =
```
$$\frac{1}{20}\left(\left(\sqrt{5}-1\right)\log\left(x^2+\frac{1}{2}\left(\sqrt{5}-1\right)x+1\right)-\left(1+\sqrt{5}\right)\log\left(x^2-\frac{1}{2}\left(1+\sqrt{5}\right)x+1\right)+4\log(x+1)-\right.$$
$$\left.2\sqrt{10-2\sqrt{5}}\,\tan^{-1}\left(\frac{-4x+\sqrt{5}+1}{\sqrt{10-2\sqrt{5}}}\right)+2\sqrt{2\left(5+\sqrt{5}\right)}\,\tan^{-1}\left(\frac{4x+\sqrt{5}-1}{\sqrt{2\left(5+\sqrt{5}\right)}}\right)\right)$$

```
=== Integrate[1/(x^6+1),x] =
```
$$\frac{1}{12}\left(-\sqrt{3}\log\left(x^2-\sqrt{3}\,x+1\right)+\right.$$
$$\left.\sqrt{3}\log\left(x^2+\sqrt{3}\,x+1\right)-2\tan^{-1}\left(\sqrt{3}-2x\right)+4\tan^{-1}(x)+2\tan^{-1}\left(2x+\sqrt{3}\right)\right)$$

```
=== Integrate[1/(x^7+1),x] =
```
$$\frac{1}{7}\sin\left(\frac{3\pi}{14}\right)\log\left(x^2+2x\sin\left(\frac{3\pi}{14}\right)+1\right)-\frac{1}{7}\sin\left(\frac{\pi}{14}\right)\log\left(x^2-2x\sin\left(\frac{\pi}{14}\right)+1\right)-$$
$$\frac{1}{7}\cos\left(\frac{\pi}{7}\right)\log\left(x^2-2x\cos\left(\frac{\pi}{7}\right)+1\right)+\frac{1}{7}\log(x+1)+\frac{2}{7}\sin\left(\frac{\pi}{7}\right)\tan^{-1}\left(\csc\left(\frac{\pi}{7}\right)\left(x-\cos\left(\frac{\pi}{7}\right)\right)\right)+$$
$$\frac{2}{7}\cos\left(\frac{3\pi}{14}\right)\tan^{-1}\left(\sec\left(\frac{3\pi}{14}\right)\left(x+\sin\left(\frac{3\pi}{14}\right)\right)\right)+\frac{2}{7}\cos\left(\frac{\pi}{14}\right)\tan^{-1}\left(\sec\left(\frac{\pi}{14}\right)\left(x-\sin\left(\frac{\pi}{14}\right)\right)\right)$$

```
=== Integrate[1/(x^8+1),x] =
```
$$-\frac{1}{8}\sin\left(\frac{\pi}{8}\right)\log\left(x^2-2x\sin\left(\frac{\pi}{8}\right)+1\right)+\frac{1}{8}\sin\left(\frac{\pi}{8}\right)\log\left(x^2+2x\sin\left(\frac{\pi}{8}\right)+1\right)-$$
$$\frac{1}{8}\cos\left(\frac{\pi}{8}\right)\log\left(x^2-2x\cos\left(\frac{\pi}{8}\right)+1\right)+\frac{1}{8}\cos\left(\frac{\pi}{8}\right)\log\left(x^2+2x\cos\left(\frac{\pi}{8}\right)+1\right)+$$
$$\frac{1}{4}\sin\left(\frac{\pi}{8}\right)\tan^{-1}\left(\csc\left(\frac{\pi}{8}\right)\left(x-\cos\left(\frac{\pi}{8}\right)\right)\right)+\frac{1}{4}\sin\left(\frac{\pi}{8}\right)\tan^{-1}\left(\csc\left(\frac{\pi}{8}\right)\left(x+\cos\left(\frac{\pi}{8}\right)\right)\right)+$$
$$\frac{1}{4}\cos\left(\frac{\pi}{8}\right)\tan^{-1}\left(\sec\left(\frac{\pi}{8}\right)\left(x-\sin\left(\frac{\pi}{8}\right)\right)\right)+\frac{1}{4}\cos\left(\frac{\pi}{8}\right)\tan^{-1}\left(\sec\left(\frac{\pi}{8}\right)\left(x+\sin\left(\frac{\pi}{8}\right)\right)\right)$$

=== Integrate[1/(x^9+1),x] =

$$\frac{1}{18}\left(-\log\left(x^2-x+1\right)+2\sin\left(\frac{\pi}{18}\right)\log\left(x^2+2x\sin\left(\frac{\pi}{18}\right)+1\right)-2\cos\left(\frac{\pi}{9}\right)\log\left(x^2-2x\cos\left(\frac{\pi}{9}\right)+1\right)+\right.$$
$$2\cos\left(\frac{2\pi}{9}\right)\log\left(x^2+2x\cos\left(\frac{2\pi}{9}\right)+1\right)+2\log(x+1)+2\sqrt{3}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)+$$
$$4\cos\left(\frac{\pi}{18}\right)\tan^{-1}\left(x\sec\left(\frac{\pi}{18}\right)+\tan\left(\frac{\pi}{18}\right)\right)+$$
$$\left.4\sin\left(\frac{2\pi}{9}\right)\tan^{-1}\left(\csc\left(\frac{2\pi}{9}\right)\left(x+\cos\left(\frac{2\pi}{9}\right)\right)\right)-4\sin\left(\frac{\pi}{9}\right)\tan^{-1}\left(\cot\left(\frac{\pi}{9}\right)-x\csc\left(\frac{\pi}{9}\right)\right)\right)$$

=== Integrate[1/(x^10+1),x] =

$$\frac{1}{40}\left(-\sqrt{10-2\sqrt{5}}\log\left(x^2-\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\,x+1\right)+\sqrt{10-2\sqrt{5}}\log\left(x^2+\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\,x+1\right)-\right.$$
$$\sqrt{2\left(5+\sqrt{5}\right)}\log\left(x^2-\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\,x+1\right)+\sqrt{2\left(5+\sqrt{5}\right)}\log\left(x^2+\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\,x+1\right)+$$
$$2\left(\sqrt{5}-1\right)\tan^{-1}\left(\frac{\sqrt{2\left(5+\sqrt{5}\right)}-4x}{1-\sqrt{5}}\right)+8\tan^{-1}(x)+2\left(1+\sqrt{5}\right)\tan^{-1}\left(\frac{4x-\sqrt{10-2\sqrt{5}}}{1+\sqrt{5}}\right)+$$
$$\left.2\left(1+\sqrt{5}\right)\tan^{-1}\left(\frac{4x+\sqrt{10-2\sqrt{5}}}{1+\sqrt{5}}\right)+2\left(\sqrt{5}-1\right)\tan^{-1}\left(\frac{4x+\sqrt{2\left(5+\sqrt{5}\right)}}{\sqrt{5}-1}\right)\right)$$