至る所微分不可能な連続関数

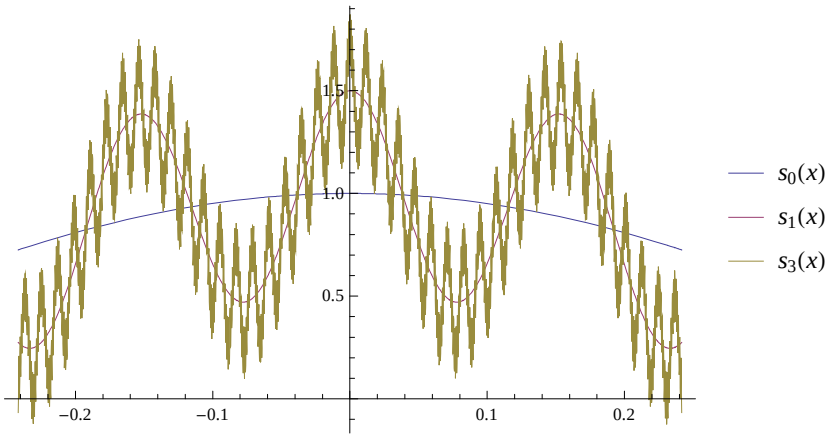
Weierstrassの関数(1875年）

関数の定義(a=1/2.b=13とする）

```
s_n_[x_] := Sum[(1 / 2)^i Cos[13^i Pi x], {i, 0, n}]
```

関数の描画

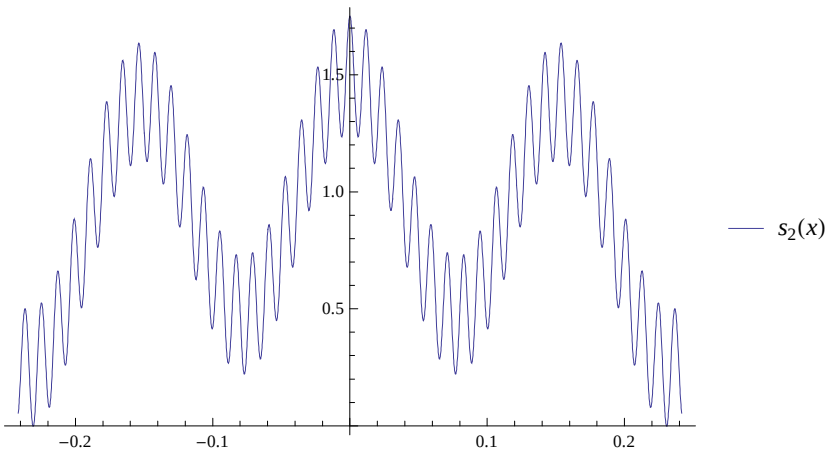
```
g1 = Plot[{s_0[x], s_1[x], s_3[x]}, {x, -Pi / 13, Pi / 13}, PlotLegends → "Expressions"]
```



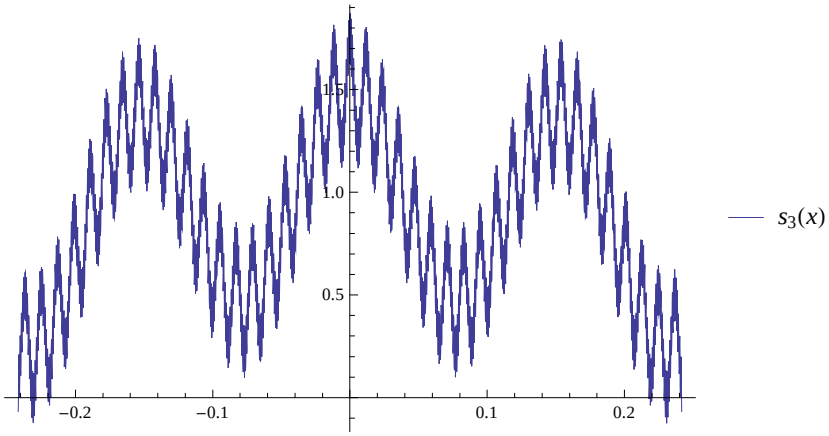

```
Export["w131220-01.gif", g1]
```

w131220-01.gif

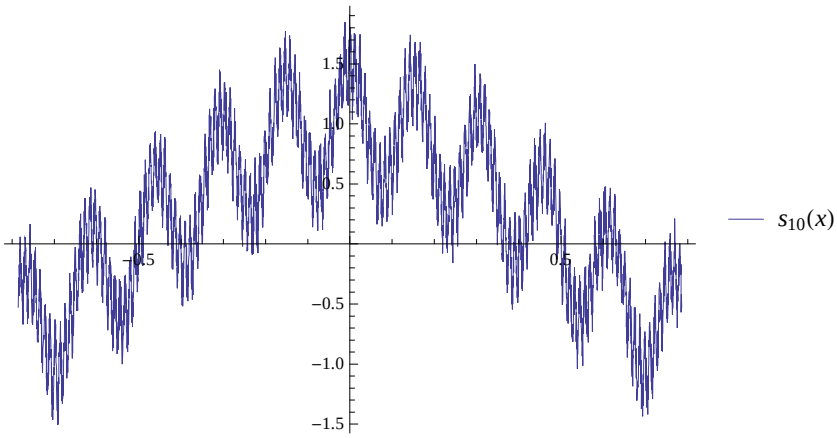
```
g2 = Plot[{s_2[x]}, {x, -Pi / 13, Pi / 13}, PlotLegends → "Expressions"]
```



```
g3 = Plot[{s3[x]}, {x, -Pi / 13, Pi / 13}, PlotLegends → "Expressions"]
```




```
g3 = Plot[{s10[x]}, {x, -Pi / 4, Pi / 4}, PlotLegends → "Expressions"]
```



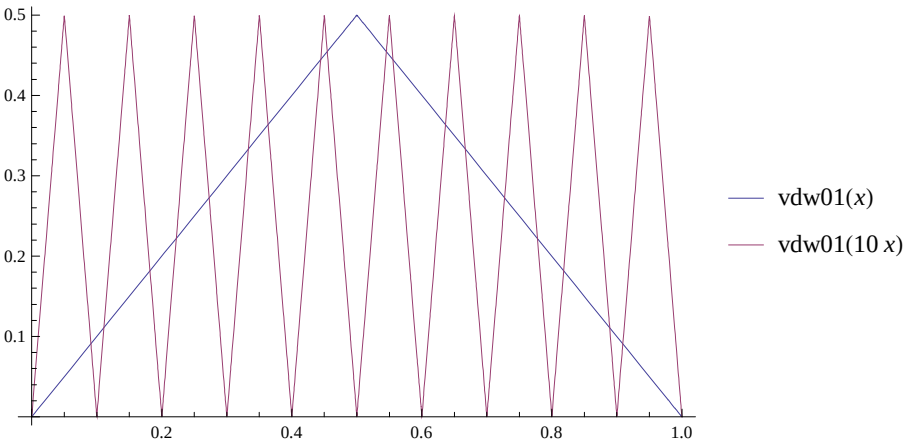
van der Waerdenの関数（1930年）

実数の10進法表記をうまくつかったもの

関数定義

```
vdw01[x_] := If[x - Floor[x] ≤ 1 / 2, x - Floor[x], Ceiling[x] - x]
```

```
Plot[{vdw01[x], vdw01[10 x]}, {x, 0, 1}, PlotLegends → "Expressions"]
```

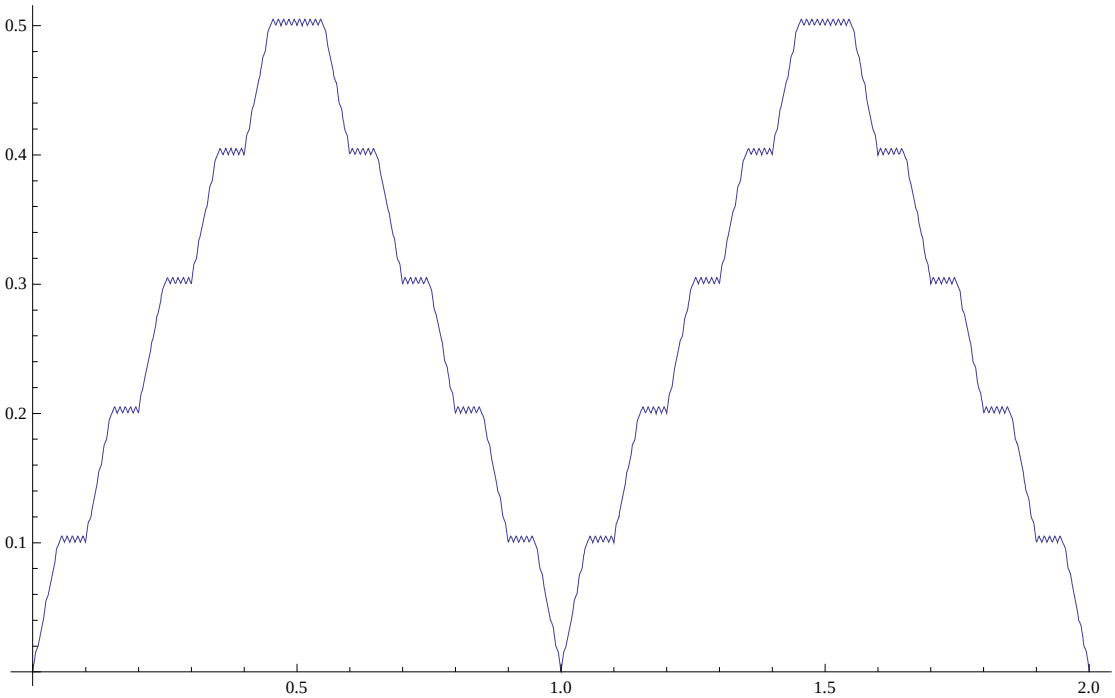


```
vdw02[x_, n_] := If[x ≥ 0, Mod[Floor[x * 10^n], 10], Mod[Floor[-x * 10^n], 10]]
```

関数のプロット

```
f[x_, m_] := Sum[N[vdw01[x * 10^n] / 10^n, 1000], {n, 0, m}]
```

```
Plot[f[x, 100], {x, 0, 2}, ImageSize → Large]
```



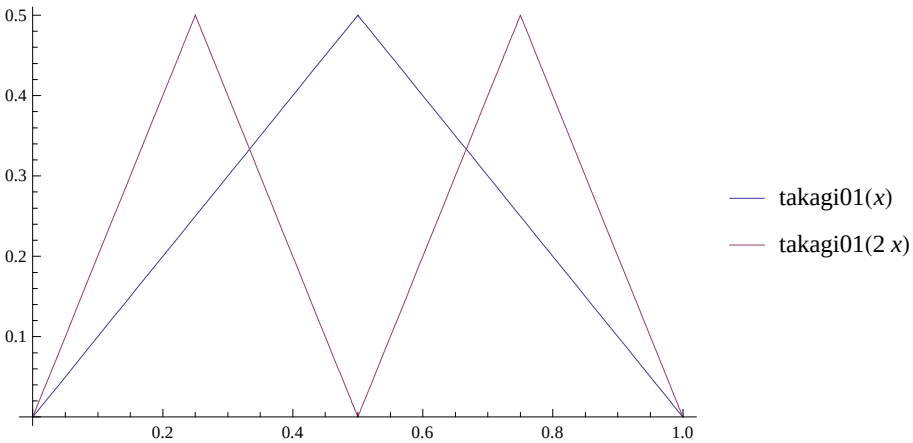
高木の関数(1903年)

```
takagi01[x_] := If[x - Floor[x] ≤ 1 / 2, x - Floor[x], Ceiling[x] - x]
```

```
takagi02[x_, n_] := If[x ≥ 0, Mod[Floor[x * 2^n], 2], Mod[Floor[-x * 2^n], 2]]
```

```
takagi03[x_, m_] := Sum[N[takagi01[x * 2^n] / 2^n, 1000], {n, 0, m}]
```

```
Plot[{takagi01[x], takagi01[2 x]}, {x, 0, 1}, PlotLegends → "Expressions"]
```



```
Plot[takagi03[x, 100], {x, 0, 2}, ImageSize → Large]
```